DYNAMIC MODEL OF GULLY EROSION

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Abstract

The main causes of gullies formation are anthropogenic factors: the clearing of native forests, tilling of fallow lands and associated change of the hydrological conditions in the rainfall-runoff system. Gully channels formation is very intensive during the period of gully initiation, when gully morphological characteristics (length, depth, width, area, volume) are far from stable .About 80 per cent of gully length, 60 per cent of its area and 35 per cent of volume are formed only at 5 per cent of gully lifetime. This stage of gully development can be described with use of the dynamic model to predict rapid changes of gully morphology.

The dynamic gully model is based on the solution of the equations of mass conservation and gully bed and walls deformations. The analysis of experimental results shows, that the rate of soil particles detachment is linearly correlated with the product of bed shear stress and mean flow velocity : In this case basic equations were written in the form of transport equation and solved with the use of explicit predictor- corrector scheme of Lax-Wendroff type. The side walls of the gully became practically straight after rapid sliding, following the incision. A straight stable slope model was used for prediction of gully side walls inclination.

This dynamic gully erosion model was verified on the data on gullies morphology and dynamics from Yamal peninsula (Russia) and New South Wales (Australia).

Introduction

The significance of gully erosion has been well documented. The volume of the gullies on the Russian Plain is about 4 10^9 m^3, i.e. about 4 per cent of the whole volume of erosion since 1700 AD (Sidorchuk, 1995). In south-east Australia with mainly pasture land the volume of gully erosion amounts to 37 per cent of whole erosion volume (Graham,1987). The gullies destroy completely the fertile topsoil layer, and the surrounding lands are damaged with more severe sheet and rill erosion.

There are two main stages of gully development, which are controlled by different sets of geomorphic processes. At the first stage of gully initiation hydraulic erosion is predominant at the gully bottom and rapid mass movement occurs on the gully sides. At the last stage of the stable gully sediment transport and sedimentation are the main processes in the gully bottom, its width increases due to lateral erosion, and slow mass movement transforms the gully sides. Gully channel formation is very intense during the period of gully initiation, when the morphological characteristics of the gully (length, depth, width, area, volume) are far from stable. This stage is relatively short and takes about 5 per cent of gully lifetime, but 80 per cent of gully length, 60 per cent of gullied area and 35 per cent of the gully volume are formed at this period (Kosov et al,1978).

The dynamic gully erosion model

The model describes the first, quick stage of gully development. At this stage the following main processes occur:

a)During the snowmelt or rainstorm event the flowing water erodes a rectangular channel in the topsoil or at the gully bottom if the flow velocity is more than critical for erosion initiation. Sediment concentration in the flow is controlled by lateral inflow from the gully catchment, detachment of the particles from the bottom and banks, and by sedimentation on the gully bottom.

b)The vertical walls of this trench are unstable. Shallow landslides transform quickly gully cross - section shape to trapezoidal at the period between adjacent water flow events.

Process of gully incision

Theoretical framework

The rate of gully erosion is controlled by water flow velocity, depth and turbulence, and soil texture, mechanical pattern and protection by vegetation. These characteristics are combined in equations of mass conservation and deformation, which can be written in the form

$$\frac{\partial Q_s}{\partial X}+\frac{\partial AC}{\partial t}=C_w q_w+M_0 W+M_b D-CV_f W \qquad (1)$$

$$-W\frac{\partial Z}{\partial t}=\frac{\partial Q_s}{\partial X}-M_b D-C_w q_w \qquad (2)$$

Here Q_s = Q C is sediment discharge (m^3/s), Q =water discharge (m^3/s); X= longitudinal coordinate (m); t=time (s); C= mean volumetric sediment concentration; A=- flow cross-section area (m^2); C_w= sediment concentration of the lateral input; q_w= specific lateral discharge; M_0 = upward sediment flux (m/s); M_b = sediment flux from the channel banks (m/s); Z=gully bottom elevations (m); W = flow width (m); D = flow depth (m); V_f = sediment particles fall velocity in the turbulent flow (m/s).

The first term in the left part of equation of mass conservation (1) defines the sediment budget in the channel reach, the second term is sediment storage in the flow . The right part of (1) defines the sediment flux: the first term is lateral flux ,the second one is upward flux, the third one is sediment flux from the banks, and the forth one is downward flux . The equation of deformation (2) defines the change of gully bottom elevation and banks coordinates according the sediment budget.

The solution of the equations with the main assumptions and simplifications

The sediment storage in the flow is usually very small and can be neglected. In this case the equation (1) is a first order ordinary differential equation, and equation (2) is a first order partial differential equation with variable coefficients. The solution of these equations depends on the form of the terms, which describe sediment fluxes.

For a given section the lateral specific discharge q_w assumed to be constant on the length L and water discharge in the flow increases linearly with the distance X from initial value Q_o: $Q=Q_o+q_w X$. The sediment concentration in the lateral flow C_w is controlled by the conditions within the basin and also assumed to be constant on the section L.

The upward sediment flux is the product of volumetric bottom sediment concentration C_0 and vertical bottom velocity of sediment particles $U\uparrow$: $M_0=\cdot U_{\uparrow}C_0$. Vertical bottom velocity of sediment particles is about 0.3 U (Rossinskiy and Debolskiy, 1980), where U (m/s) is mean flow velocity. The near bed sediment concentration (or probability of particles detachment) after H.Einstein (1942) is function of the measure of the transport rate $C_0 = f_1(\tau/\tau_{cr})$. Here $\tau = g\rho DS$ is the bed shear stress, τ_{cr} is its value for sediment detachment initiation , g - acceleration due to gravity (m/s^2), ρ - water density (kg/m^3), S - flow surface slope (for gullies is equal to bed slope).

Mirtskhulava(1988) showed, that critical shear stress is mainly controlled by forces of friction and cohesion: $\tau_{cr} = 1.2\lambda\ (m/n)\left[(\rho_s - \rho)gd + 1.25C_f^n K\right]$. Here λ is coefficient of flow resistance: $\lambda = 0.18*(d/D)^{1/3}$; m is equal to1. for clean water flows, and is equal 1.4 for the flows with colloidal particles content more than 0.1 kg/m^3; parameter of turbulence n is usually about 4; ρ_s is sediment density (kg/m^3); d - mean diameter of soil aggregates (m); K - coefficient of variability of soil mechanical pattern, usually it is 0.5; $C_f^n = f_2(C_h)$ is soil fatigue strength to rupture and it is the function of soil cohesion C_h (Pa).The first term in square brackets represents the influence of friction on particle stability, and is of the main importance for noncohesive soils, the second term represents the influence of cohesion on particle stability, and is of the main importance for cohesive soils.

Field experiments were run to determine the functions $C_0=f_1(\tau/\tau_{cr})$ and $C_f^n = f_2(C_h)$ for gully erosion conditions. Flumes 1, 2 and 3 being 9.7, 3.5 and 6.0 m long respectively were prepared in natural soils on the sides of Brook gully (Yass River basin, NSW, Australia) with different inclination . The water entered the head of the flume from reservoir (tank) with a volume of 15 m^3 through a transportable weir, that permitted constant discharge up to 12 l/s non less than 15 min. The water samples for sediment concentration determination were taken several times during each run at the head and the end of the flume to estimate sediment budget $\Delta Q_s/\Delta X$ along the flume and particle detachment rate $M_0 = (1/W)*\Delta Q_s/\Delta X$ (sedimentation and bank erosion was negligible). The main hydraulic parameters of the flow were measured during the run, and soil cohesion was measured with torevane after each run (tab.1). The soil was composed mainly with silt particles and had cohesion 3.0-7.0 10^4 Pa at saturation.

flume	run	Q	U	W	D	S	C_h	$(1/W)dQ_s/dX$	k_eUDS
		m^3/s	m/s	m	m		Pa 10^5	m/s 10^{-5}	m/s 10^{-5}
1	1	0.0014	0.420	0.180	0.019	0.063	0.29	0.132	0.151

1	2	0.0022	0.370	0.170	0.034	0.063	0.29	0.236	0.243
1	3	0.0042	0.580	0.177	0.041	0.063	0.29	0.513	0.458
1	5	0.0086	0.640	0.208	0.064	0.068	0.29	0.599	0.847
1	6	0.0078	0.600	0.220	0.060	0.064	0.29	0.704	0.695
2	3	0.0015	0.534	0.120	0.023	0.285	0.31	0.936	0.962
2	4	0.0023	0.644	0.130	0.028	0.285	0.31	1.100	1.370
2	5	0.0041	0.587	0.165	0.042	0.240	0.31	0.919	1.590
2	6	0.0057	0.641	0.190	0.047	0.240	0.31	2.870	1.950
2	7	0.0086	0.730	0.233	0.034	0.248	0.31	2.220	2.460
2	9	0.0110	1.127	0.277	0.035	0.252	0.31	4.050	2.700
3	3	0.0011	0.830	0.120	0.008	0.577	0.50	0.571	0.577
3	4	0.0015	1.726	0.132	0.007	0.578	0.52	0.555	0.680
3	5	0.0019	1.709	0.142	0.008	0.579	0.52	0.747	0.787
3	6	0.0023	1.817	0.151	0.009	0.580	0.53	0.880	0.892
3	7	0.0032	1.700	0.195	0.009	0.582	0.54	1.060	0.903
3	8	0.0041	1.900	0.177	0.012	0.584	0.59	0.700	1.100
3	9	0.0049	1.900	0.195	0.013	0.588	0.61	1.130	1.130
3	10	0.0063	0.720	0.202	0.015	0.590	0.64	1.360	1.280
3	11	0.0083	0.800	0.226	0.018	0.594	0.74	1.680	1.140

Table 1

The parameters of the flow , sediment budget and soils in experimental flumes in the Brook gully (Yass River basin, NSW,Australia)

The analysis of experiment results shows, that in the conditions of steep slopes, common for gullies, the rate of soil particles detachment is linearly correlated with the product of bed shear stress and mean flow velocity :

$$M_0 = k_e UDS \qquad (3).$$

This formula was also validated on the basis of field experiments in the gullies of Yamal peninsula (north of the Western Siberia, Russia) and showed satisfactory conformity to these data (Sidorchuk, in press). The erodibility coefficient k_e equals to 6.46 $10^{-2}/\tau_{cr}$. For calculation of the critical bed shear stress the formula of Mirtskhulava can be used with $C_f^n = 6.7*10^{-7} C_h^2$. The latter expression is based on limited set of data and additional experiments must be produced for its verification.

The width of the flow in gullies can be calculated with the empirical formula: $W = 0.3Q^{0.4}$ (based on data from Yamal peninsula), and depth and velocity with Chezy formula.

The process of flow bank erosion in the gullies has not been satisfactorily investigated. It is assumed that rate of bank erosion dW_b/dt is equal to sediment flux from the banks M_b . Using an analogy with estimations of the river bank erosion the expression $M_b = M_0 * V / U$ can be suggested as the first approximation. Here V is lateral velocity, W_b is gully bottom width. In large canals M_b is usually about 5 per cent of M_0 (Vikulova,1972). For a curved channel Rozovzkiy (1957) obtained a simple formula: $V = 11.0 * U * D / R$. At the narrow incised gully bottom with W_b < 10.0W the radius R of confined meanders will decrease when W_b increases due to banks erosion: R=50.0W(W/W_b). When W_b became > 10.0W the flow forms free meanders with R=5.0W. At the same time curved flow can wash only part of side walls and this part P_e decreases when the relative bottom width increases. The investigations in the gullies of Yamal peninsula shows, that P_e = W/W_b . After combining all these formulas the expression for calculation of gully bank erosion rate takes the form: $\frac{dW_b}{dt} = k_b * M_0$. Here $k_b = 0.22 * D / W$. when $W_b < 10.0 * W$ and $k_b = 2.2 * D / W_b$. when $W_b > 10.0 * W$.

The expression for downward flux is rather simple and includes the product of fall velocity in turbulent flow and depth-averaged sediment concentration in flow. The fall velocity in the turbulent flow is lower, than Stocks fall velocity in laminar flow or in steady water V_{st}, and in the case of thin particles and high turbulence can be 0.

After substitution of equations (1) and (3) into (2) it takes the form of transport equation in terms of bottom elevations Z:

$$\frac{\partial Z}{\partial t} - a\frac{\partial Z}{\partial x} - V_f C = 0. \qquad (4)$$

Here a $=k_e UD$. The equation (4) can be numerically solved with aim of explicit predictor-corrector scheme of Lax-Wendroff type

$$Z_i^{j+1/2} = (1-\beta\ )Z_i^j + \beta\ Z_{i+1}^j - \alpha\ \frac{\Delta t}{\Delta x}\left[\frac{(aq)_{i+2}^j + (aq)_{i+1}^j}{2} Z_{i+1}^j - \frac{(aq)_{i+1}^j + (aq)_i^j}{2} Z_i^j\right]$$

$$Z_i^{j+1} = Z_i^j - \frac{\Delta t}{2\alpha\ \Delta x}\{\ (\alpha-\beta\ )\frac{(aq)_{i+2}^j + (aq)_{i+1}^j}{2} Z_{i+1}^j - (2\beta-1)\frac{(aq)_{i+1}^j + (aq)_i^j}{2} Z_i^j +$$

$$(1-\alpha-\beta\ )\frac{(aq)_i^j + (aq)_{i-1}^j}{2} Z_{i-1}^j + \frac{(aq)_{i+1}^j + (aq)_i^j}{2} Z_i^{j+1/2} - \frac{(aq)_i^j + (aq)_{i-1}^j}{2} Z_{i-1}^{j+1/2}\ \} + V_f C_i \Delta t$$

The symbol 'i' represents the change by the length, symbol 'j ' - in time. For the sediment concentration C_i the solution of (1) on the flow reach with length Δx will be used:

$$C_i = \left(C_{i-1} - \frac{(k_e + k_b)*Q_{i-1}S}{q_w(Y+1)} - \frac{C_w}{Y}\right)*\left(\frac{Q_{i-1}}{Q_i}\right)^Y + \frac{(k_e + k_b)*Q_i\ S}{q_w(Y+1)} + \frac{C_w}{Y}$$

Here C_o - sediment concentration in the channel flow at the beginning of the reach, $Y = \left(q_w + V_f * W\right)/q_w$. The best fit values of net numbers α and β are: β = 0.75--1.0; α. =0.25--0.5. For the explicit scheme stability the Courant number must be less than 1.0: $aq\,\Delta t/\Delta x \le 1$. The same approach was used for numerical solving of equations of bank erosion rate.

The process of the side walls transformation

The side walls of the gully becomes practically straight after rapid sliding, following the incision. In this case a straight stable slope model can be used for prediction of gully side wall inclination. If the depth of incision D_v is more than $D_v = \frac{2.0*C_h}{g*\rho_s}\cos(\varphi)\Big/\sin^2\frac{1}{2}\left(\varphi + \frac{\pi}{2}\right)$, then gully walls inclination ϕ can be calculated with the help of formula: $\frac{C_h}{g*\rho_s*D_v} = \frac{\rho_s - w*\rho}{\rho}\tan(\varphi)*\cos^2(\phi\ ) - \frac{\sin(2\phi\ )}{2}$. Here w is volumetric water content in the soil, φ is the angle of internal friction.

When the bottom width, wall inclination and volume of incision V_0 are known, the shape of the gully cross-section can be transformed into a trapezium with bottom width W_b, depth $D_t = \left(\sqrt{W_b^2 + \frac{4V_0}{\tan(\phi)}} - W_b \right) \frac{\tan(\phi)}{2}$ and top width $W_t = W_b + 2.0 D_t / [\tan(\phi)]$.

Algorithm of dynamic model of gully erosion

The input to the model includes topographical, hydrological and lithological data. Topography is described by elevations and distances from the gully mouth in N points of the longitudinal profile of each flowline on initial slope (including existing gullies). The water discharge change in time (hydrograph) has to be calculated for all these points with the hydrological model (which must be linked with the gully erosion model). The multilayer soil properties are used in the model, for each layer an input is needed for elevations of the base of the layer in the same N points; soil density; cohesion; angle of internal friction; stable aggregates diameter; water content; thin vegetation roots content.

The longitudinal profile transformation in space and time and gully bottom widening are calculated with Lax -Wendroff predictor-corrector scheme. The stability criterion is determined for each calculation step. The numerical scheme stability is attained by change of time step duration . On each step by the length the flow width, depth , velocity and critical shear stress for the soils in the bottom and in the banks are calculated. After each flood event the rectangular bottom trench is transformed with straight slope model to trapezoidal shape and longitudinal distribution of the gully width (top and bottom), depth and bottom elevations are estimated.

Dynamic model verification

The model was used for prediction of gully erosion on the Yamal peninsula (Sidorchuk, in press), and of Brook gully in the Yass River basin, NSW, Australia. Table 2 shows calculated with the model and observed elevations of longitudinal profile of Brook gully. The erodibility coefficient k_e was determined from the data in table 1, and equals to 2.0 10^{-3} for the loam soils at the upper section of the gully. The gully profile in 1932 was reconstructed from the plan of lot 64 in Parish Purrorumba, County of Murray and aerophotoes of 1941, and elevations of 1992 were measured during the field works. The runoff for period 1932-92 was calculated from River Yass discharge data. The comparison of calculated and measured longitudinal profiles is satisfactory for lower part of incision into pebbly loams. At the upper part of this section the observed incision was more than predicted due to seepage from the pond at the head of gully. a calibrated model . Numerical experiments show, that the model is sensitive to change of the erodibility coefficient value, and field investigations and careful calibration of the model are necessary for accurate prediction of gully erosion.

Distance from the gully mouth (m)	Observed elevations of longitudinal profile in 1932 (m from MSL)	Calculated elevations of longitudinal profile in 1988 (m from MSL)	Observed elevations of longitudinal profile in 1988 (m from MSL)	Soil texture
.00	683.80	683.80	683.80	schist
38.82	686.60	686.60	686.60	schist
232.90	688.70	688.70	688.70	schist
349.34	692.30	692.30	693029	schist
426.98	695.10	695.10	695.10	schist
582.24	697.50	697.50	697.50	schist
659.87	698.70	698.70	698.70	schist
737.50	699.70	699.50	699.50	schist
776.32	700.70	699.60	700.20	schist
815.14	702.40	699.60	700.60	pebbly loam
853.95	704.00	700.80	701.30	pebbly loam
892.77	704.40	701.60	701.90	pebbly loam
931.58	704.80	702.50	702.60	pebbly loam

970.40	705.10	703.30	703.30	pebbly loam
1009.22	706.50	705.70	704.70	pebbly loam
1164.48	706.90	706.30	704.90	pebbly loam
1203.30	707.30	706.90	705.10	pebbly loam
1280.93	707.90	707.80	705.60	pebbly loam
1319.74	709.80	709.20	709.20	pebbly loam
1358.56	712.00	712.00	712.00	pebbly loam

Table 2. Calculated and observed deformation of the longitudinal profile of Brook gully (River Yass basin, NSW,Australia)

Conclusion

The dynamic gully erosion model describes the first, quick stage of gully development. During the snowmelt or rainstorm event the flowing water erodes a rectangular channel in the topsoil or at the gully bottom. Change of the gully bottom elevations is controlled mainly by upward detachment of the particles from the bed and by sedimentation on the gully bottom. This process is described by transport equation $\frac{\partial Z}{\partial t} - k_e UD \frac{\partial Z}{\partial x} - V_f C = 0$, which is numerically solved with aim of explicit predictor- corrector scheme of Lax-Wendroff type. The vertical walls of this channel are unstable. At the period between adjacent water flow events shallow landslides transform quickly gully cross - section shape to trapezoidal with bottom width W_b, depth $D_t = \left(\sqrt{W_b^2 + \frac{4V_0}{\tan(\phi)}} - W_b \right) \frac{\tan(\phi)}{2}$ and top width $W_t = W_b + 2.0 D_t / \left[\tan(\phi) \right]$. Numerical experiments show, that the model is sensitive to change of the coefficient k_e value, and field investigations and careful calibration of the model are necessary for accurate prediction of gully erosion.

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